OCR
Oxford Cambridge and RSA

…day June 20XX – Morning/Afternoon

AS Level Further Mathematics A

Y534 Discrete Mathematics

SAMPLE MARK SCHEME

Duration: 1 hour 15 minutes

MAXIMUM MARK 60

This document consists of 16 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for AS Level Further Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some papers. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

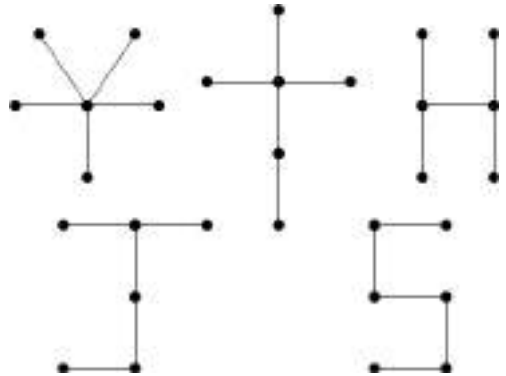
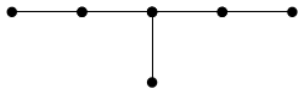
i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

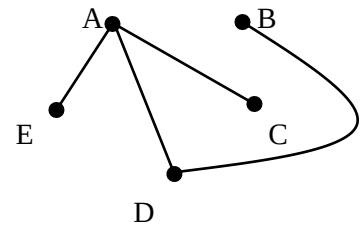
Question			Answer	Marks	AO	Guidance	
1			(A) e.g. Is this journey possible? e.g. Is it possible to travel from Edinburgh to Southampton, leaving Edinburgh after 9 am and arriving in Southampton by 4 pm?	**B1**	**1.1**	Any relevant problem that demonstrates the idea of existence	In the context of this train journey
1			(B) e.g. What is the latest time I can leave Edinburgh to arrive in Southampton by 4 pm?	**B1** **[2]**	**1.1**	Any relevant problem that demonstrates the idea of optimisation	In the context of this train journey
2	**(i)**		3 possible orders: ABCD ACBD CABD	**B1** **[1]**	**1.1**	Stating 3 or listing the three orders	
2	**(ii)**		E must come after C, but otherwise it can go anywhere	**B1**	**1.1**	Seen, or implied by being true for all given orders	
			If A, B, C, D are in the order A B C D then E can go immediately after C or D $\Rightarrow 2$ If they are in the order A C B D then E can go immediately after C, B or D $\Rightarrow 3$ If they are in the order C A B D then E can go immediately after C, A, B or D $\Rightarrow 4$	**M1**	**2.1**	Breaking the problem up into cases	A B C E D A B C D E A C E B D A C B E D A C B D E C E A B D C A E B D C A B E D C A B D E
			In total there are 9 possible orders	**E1** **[3]**	**2.4**	Showing that there are 9 orders and no more	

Question			Answer	Marks	AO	Guidance	
2	(iii)		F must come after C	B1	1.1	Seen, or implied by being true for all given orders	
			If C is in the third position (A B C E D, A B C D E) there are 3 possibilities for F $\Rightarrow 6$ If C is in the second position (A C E B D, A C B E D, A C B D E) there are 4 possibilities for F $\Rightarrow 12$ If C is in the first position (C E A B D, C A E B D, C A B E D, C A B D E) there are 5 possibilities for F $\Rightarrow 20$	M1	2.1	Making a substantial start at counting the possibilities, this may involve starting again	A listing of the possibilities, without written reasoning, would score **M0**
			In total there are 38 possible orders	E1 [3]	2.4	With appropriate working	
3	(i)		Let X play strategy P with probability p and strategy Q with probability $1-p$ If Y plays strategy R, X can expect to win $4p-3(1-p)=7p-3$	M1	3.3	Calculating expected winnings if Y plays R or S	May use p for $\mathrm{P}(X \text{ plays } Q)$
			If Y plays S, X can expect $-2p+(1-p)=1-3p$	A1	1.1	Both correct	May add a constant throughout Need not simplify expressions
			$p=0 \Rightarrow \min \mathrm{E}(\text{win})=-3$ $p=1 \Rightarrow \min \mathrm{E}(\text{win})=-2$	E1	3.4	Explicitly considering extreme points, or using a sketch graph to show that optimum point is at intersection	
			$7p-3=1-3p \Rightarrow p=0.4$ and $\min \mathrm{E}(\text{win})=-0.2$	M1	1.1a	Solving their expressions simultaneously to achieve their p	BC
			X should choose randomly between strategies P and Q so that P has prob 0.4 and Q has prob 0.6	A1 [5]	3.4	Interpretation of $p=0.4$ in context	

Question			Answer	Marks	AO	Guidance	
3	(ii)		E.g. You must assume that *Y* is not going to play predictably E.g. You must assume that *Y* is not going to use a pure strategy	**E1** **[1]**	**3.5b**	For one limitation of the model with reference to *Y*'s limited behaviour	
4	(i)		Route e.g. $c – b – a – e – d – c – b – e – d$	**B1** **[1]**	**1.1**	A route that starts and ends at *c* and *d* and uses every edge once and only once	
4	(ii)		It has exactly two odd vertex orders	**B1** **[1]**	**1.2**	Two and only two odd	
4	(iii)		In graph 1 there are two arcs directly joining *b* to *c* (or *d* to *e*) In graph 2 there are two arcs directly joining *v* to *w*	**B1** **B1** **[2]**	**1.1** **1.1**	Reason why graph 1 is not simple Reason why graph 2 is not simple	
4	(iv)		e.g. in graph 1 the vertex of degree 2 (*a*) is adjacent to each vertex of degree 4 (*b* and *e*), whereas in graph 2 the vertex of degree 2 (*y*) is adjacent to one vertex of degree 4 (*z*) but not the other (*w*) e.g. graph 1 has two pairs of vertices directly joined by two arcs (*bc* and *de*) whereas graph 2 only has one such pair (*vw*)	**M1** **A1** **[2]**	**2.3** **1.1**	Partially correct explanation of why the graphs are not isomorphic Fully correct explanation	

Question			Answer	Marks	AO	Guidance	
5	(i)			B1 [1]	1.1	These three graphs and no others	May appear in any equivalent variation
5	(ii)		e.g. Six vertices so need 6 positive integers e.g. 5 arcs so sum of vertex orders $=10$ e.g. Tree must have at least two 'ends' so at least two vertex orders have value 1	B3 [3]	1.1 1.1 2.5	**B1** for each independently correct property	Award **B2** for three properties if incorrect or inconsistent terminology used
5	(iii)		$\{1, 1, 1, 1, 1, 5\}$ $\{1, 1, 1, 1, 2, 4\}$ $\{1, 1, 1, 1, 3, 3\}$ $\{1, 1, 1, 2, 2, 3\}$ $\{1, 1, 2, 2, 2, 2\}$	M1 A1 [2]	1.1 1.1	At least four correct sets All five correct with no extras	
5	(iv)			M1 A1 [2]	1.1 1.1	At least four correct graphs Five correct graphs and no extras, unless it is because both versions of $\{1, 1, 1, 2, 2, 3\}$ are shown	Other possibility for $\{1, 1, 1, 2, 2, 3\}$ 

Question			Answer	Marks	AO	Guidance	
6	(i)		Total mass is 60, so each bin must hold at least 20kg	B1	1.1	Consider total mass to find a lower bound for capacity	
			If it is 20kg then using first-fit: 8 5 7 9 7 1 3 9 3 8	M1 A1	3.1b 1.1	Attempt first-fit with a capacity of at least 20kg All correct, in this order	First bin correct, in this order
			Using first-fit decreasing: 9 9 1 8 8 3 7 7 5 3	M1 A1	3.1b 1.1	Attempt first-fit decreasing with a capacity of at least 20kg All correct, in this order (or a description in words)	Correct placement of the values 9 9 8 8 7 7
			But if the capacity was 21kg then the 3 could go into the first bin and the 1 into the second bin, so only three bins are needed				
			So the bins cannot be more than 20kg, so they must be exactly 20kg	E1	3.2a	20 is also the upper bound, so the capacity is 20	
				[6]			
6	(ii)		Shuttle sort has quadratic order, as a function of the length of the list, so the time to run first-fit decreasing would be the sum of a quadratic function and a linear function, which is a quadratic function	M1	1.1	Knowing and using the fact that shuttle sort has quadratic order	$T(n) = O(n^2) + O(n)$
			This means that first fit-decreasing has quadratic order in this case	A1	2.2a		$= O(n^2)$
				[2]			

Question			Answer	Marks	AO	Guidance	
7	(i)		e.g. (5) in C, (4) in D, (2) in E Using Prim's algorithm starting at A $AE = 2$ $AD = 4$ $AC = 5$ (or DB) $DB = 5$ (or AC) Total weight $= 16$  	**M1** **A1** **A1** **B1** **[4]**	**1.1a** **1.1** **1.1** **1.1**	Stating which is the starting vertex A valid order of building the tree for their starting vertex, clearly shown (arcs or vertices with arcs indicated on matrix) Correct (labelled) tree Weights need not be shown	
7	(ii)	(a)	Vertex A	**B1** **[1]**	**3.1a**		
	(ii)	(b)	$0+(4+5)+5+4+2$ $\Rightarrow 20$	**B1** **[1]**	**1.1**		

Question			Answer	Marks	AO	Guidance	
7	(iii)		$(1\times4-1)+(2\times3-1)+(3\times2-1)+(4\times1-1)$	M1	2.1	1×4, 2×3, etc.	Or by referring to their working in part **(i)**
				M1	1.1	Subtracting 1 for each pass	
			$=3+5+5+3=16$	A1	1.1	www	
				[3]			
7	(iv)		Cubic order	M1	1.1	Stating or using the fact that Prim's has cubic order	
			Approximately $\left(\frac{500}{100}\right)^3\times4=500$ seconds oe	A1	2.2b	Must include an indication that this is an approximation	8 minutes 20 seconds
				[2]			

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th><th></th></tr>
<tr><td>8</td><td>(i)</td><td></td><td>5 milk chocolate in each type, $4\times5=20$</td><td>B1

[1]</td><td>2.2a</td><td>Using numbers of milk chocolate truffles</td><td></td></tr>
<tr><td>8</td><td>(ii)</td><td></td><td>2 so that there are at least 12 nutty chocolate truffles</td><td>B1

[1]</td><td>2.2a</td><td>2 (Assorted boxes) and using numbers of nutty chocolate truffles</td><td></td></tr>
<tr><td>8</td><td>(iii)</td><td></td><td>
<table>
<tr><td colspan="3">Type</td><td>Cost</td></tr>
<tr><td>Assorted</td><td>No Nuts</td><td>Speciality</td><td>£</td></tr>
<tr><td>4</td><td>0</td><td>0</td><td>8.00</td></tr>
<tr><td>3</td><td>1</td><td>0</td><td>7.00</td></tr>
<tr><td>3</td><td>0</td><td>1</td><td>8.50</td></tr>
<tr><td>2</td><td>1</td><td>1</td><td>7.50</td></tr>
<tr><td>2</td><td>0</td><td>2</td><td>9.00</td></tr>
</table>

3× Assorted and 1× No Nuts</td><td>M1

A1

B1
[3]</td><td>1.1

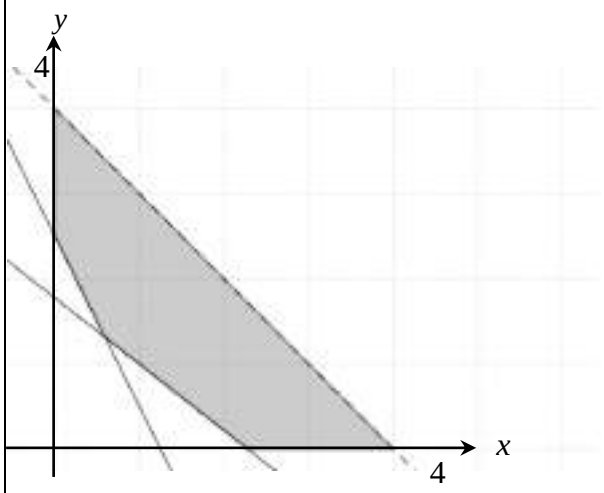
1.1

1.1</td><td>Identifying at least three of these five cases for the number of boxes of the different types bought

All five cases, and no others, unless explicitly rejected as being infeasible

cao</td><td>Need not show costs, and if given do not penalise calculation errors

May be marked in list</td></tr>
</table>

Question			Answer	Marks	AO	Guidance	
8	(iv)	(a)	P: $8x+4y\geq 10$ oe W: $7x+9y\geq 16$ oe $x\geq 0$ and $y\geq 0$ (and integer-valued)	**M1** **A1** **[2]**	**3.1b** **1.2**	One of $8x+4y\geq 10$ and $7x+9y\geq 16$, in any form Both of these correct, in any form	Allow strict inequalities, but not an equality or with inequality reversed
8	(iv)	(b)		**M1** **A1** **[2]**	**3.3** **1.1**	Plotting lines and identifying feasible region All lines and feasible region correct on scaled axes	For reference: (0, 2.5) (0.59, 1.32) or better (2.28 to 2.29, 0)
8	(iv)	(c)	(0, 3) + 2 small milk + 2 small nutty $\Rightarrow$ £9.50 (1, 2) + 2 small milk + 2 small nutty $\Rightarrow$ £8.00 (2, 1) + 2 small milk + 3 small nutty $\Rightarrow$ £7.00 (3, 0) + 2 small milk + 3 small nutty $\Rightarrow$ £5.50 (1, 1) + 3 small milk + 3 small nutty $\Rightarrow$ £6.50 Cheapest solution costs Narendra £5.50	**M1** **M1** **E1** **[3]**	**3.4** **3.2a** **3.4**	Calculating costs for any integer-valued feasible solution Including sufficient small packs to deal with 'milk' and 'nutty' requirements £5.50	May go straight to solution (from logical reasoning or calculation) **E0** if exterior points used as if feasible **E0** if correct answer given with insufficient evidence to justify it

Assessment Objectives (AO) Grid

Question	AO1	AO2	AO3(PS)	AO3(M)	Total
1	2				**2**
2(i)	1				**1**
2(ii)	1	2			**3**
2(iii)	1	2			**3**
3(i)	2			3	**5**
3(ii)				1	**1**
4(i)	1				**1**
4(ii)	1				**1**
4(iii)	2				**2**
4(iv)	1	1			**2**
5(i)	1				**1**
5(ii)	2	1			**3**
5(iii)	2				**2**
5(iv)	2				**2**
6(i)	3		3		**6**
6(ii)	1	1			**2**
7(i)	4				**4**
7(ii)	1		1		**2**
7(iii)	2	1			**3**
7(iv)	1	1			**2**
8(i)		1			**1**
8(ii)		1			**1**
8(iii)	3				**3**
8(iv)(a)	1		1		**2**
8(iv)(b)	1			1	**2**
8(iv)(c)			1	2	**3**
Totals	**36**	**11**	**6**	**7**	**60**

PS = Problem Solving
M = Modelling

Summary of Updates

Date	Version	Change
October 2019	2	Amendments to the front cover rubric instructions to candidates

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OCR
Oxford Cambridge and RSA

AS Level Further Mathematics A
Y534 Discrete Mathematics
Printed Answer Booklet Version 2

Date – Morning/Afternoon
Time allowed: 1 hour 15 minutes

You must have:
- Question Paper Y534 (inserted)
- Formulae AS Level Further Mathematics A

You may use:
- a scientific or graphical calculator

First name									
Last name									
Centre number						**Candidate number**				

INSTRUCTIONS
- The Question Paper will be found inside the Printed Answer Booklet.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

1(A)	
1(B)	
2(i)	
2(ii)	
2(iii)	

3(i)	
3(ii)	
4(i)	
4(ii)	
4(iii)	
4(iv)	

5(i)	
5(ii)	
5(iii)	

5(iv)

6(i)

6(ii)	

PLEASE DO NOT WRITE IN THIS SPACE

7(i)

	A	**B**	**C**	**D**	**E**
A	-	9	5	4	2
B	9	-	7	5	7
C	5	7	-	6	8
D	4	5	6	-	5
E	2	7	8	5	-

7(ii)

7(iii)

7(iv)

8(i)	
8(ii)	
8(iii)	

8(iv)(a)

8(iv)(b)

8(iv)(c)

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